# Field theoretical methods in condensed matter

## Problem set 5

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1. **Lehmann Representation**

   Consider the spectral function in the Lehmann representation

   $$A(\omega) = \frac{2\pi}{\mathcal{Z}} \sum_{n,m} X_1^{nm} X_2^{mn} \left[e^{-\beta E_n} \pm e^{-\beta E_m}\right] \delta(\omega + E_n - E_m) \tag{1}$$

   with $[X_1, X_2]_\pm = 1$. Prove the following sum rule:

   $$1 = \int \frac{\mathrm{d}\omega}{2\pi} A(\omega)\,. \tag{2}$$

1. **Spectral Functions**

   For the Hamiltonian of problem sheet 3,

   $$H = -t \sum_{j=1}^{N} \left[c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right] - \mu \sum_{j=1}^{N} c_j^\dagger c_j\,, \tag{3}$$

   calculate the spectral function.
   *Hint: Momentum may be a more convenient basis!*

2. **Spectral Representation**

   In the lecture the spectral representation of the retarded Green's function was calculated. Write down the equivalent for the time ordered and advanced Green's functions. How are the real and imaginary parts of the three Green's functions related? What are their analytical properties in the complex plane (as a function of $\omega$)?