Field theoretical methods in condensed matter

Problem set 4

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1. Jordan-Wigner transformation

In the lecture we have introduced the Jordan-Wigner transformation

$$S_j^z \to n_j - 1/2, \quad S_j^+ \to (-1)^j c_j^\dagger \mathrm{e}^{i\pi\phi_j}, \quad S_j^- \to (-1)^j c_j \mathrm{e}^{-i\pi\phi_j}$$

between the spin operator $\vec{S}_j$ and annihilation (creation) operators $c_j^{(\dagger)}$ of *spinless* fermions at site j. Here $n_j = c_j^\dagger c_j$ and $\phi_j = \sum_{l=1}^{j-1} n_l$.

a) Show that the c_j fulfill an anti-commutation algebra if the spins fulfill an angular momentum algebra!

b) Rewrite the interaction

$$H' = J' \sum_j \vec{S}_j \cdot \vec{S}_{j+2}$$

in terms of the spinless fermions c_j!

c) Can we also use the Jordan-Wigner transformation for Heisenberg models in two and three dimensions?

2. Ising chain with open boundaries

In the lecture we have studied the exact solution of the Ising chain with magnetic field for periodic boundary conditions using transfer matrices. Repeat this calculation for the case of open boundaries, i.e., an Ising chain of length L with spins σ_1, respectively σ_L, at the ends of the chain. Calculate, in particular,

a) the partition function Z and the free energy f. How does the result compare to the periodic case in the *thermodynamic limit*?

b) the correlation function $\langle \sigma_i \sigma_j \rangle$. What is fundamentally different now from the periodic case?