Field theoretical methods in condensed matter

Problem set 1

(To be solved and discussed in class)

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1. **Fourier transformation**

   In the lecture we considered the quantum harmonic chain of length L in the continuum limit with periodic boundary conditions. For the quantized field describing the distortions of the chain $\hat{\phi}(x)$ we introduced the Fourier transform
   $$\hat{\phi}(x) = \frac{1}{L}\sum_k \mathrm{e}^{ikx}\hat{\phi}_k$$
   and the back transform
   $$\hat{\phi}_k = \frac{1}{L}\int_0^L dx\, \mathrm{e}^{-ikx}\hat{\phi}(x).$$
   Derive the quantization condition for k and show that the two transformations are consistently defined.

2. **Open boundary conditions**

   Now we want to consider the same chain but with open boundary conditions $\hat{\phi}(0) = \hat{\phi}(L) = 0$.
   What does the Fourier transformation look like in this case and what is the quantization condition for the wave vector k?

3. **Euler-Lagrange equations**

   Consider the classical action (Wirkung) for the field ϕ:
   $$S[\phi] = \int dt\, L[\phi] = \int dt\, dx\, \mathcal{L}(\phi, \partial_t\phi, \partial_x\phi).$$
   Derive the Euler-Lagrange field equations using Hamilton's principle of least action.